

AN EXAMINATION OF A
SURVEILLANCE-EVASION MODEL

By

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THESIS

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ABSTRACT

This study examines a differential game surveillance-evasion model in an effort to evaluate an existing model, bridge the gap between theory and applications of the theory and to attempt to provide insight into further extension of theory.

Escape paths generated by the model were plotted for the case where surveillance could not be maintained. Shortcomings of the model, most notably the lack of provision for the reacquisition of the Evader after escape, were discussed and suggestions for future models to accomplish this were made.

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I. INTRODUCTION

This report documents the results of an examination of a differential game model of a surveillance-evasion situation as related to Naval Warfare. Particular attention was paid to the aspect of adequacy of the model to realistically depict the proper operational situation and to the applicability of the model to the operational requirements currently required to be met by the Naval forces at sea.

The particular model which was the subject of this evaluation was one used by J. Taylor [1]. This work was undertaken in an effort to bridge the gap between pure theory and application and to provide insight into possible future extensions of theory.

II. DESCRIPTION OF THE MODEL

The model presented in [1] is summarized here for convenience of discussion.

A. STATEMENT OF THE PROBLEM

A tracker, or pursuer, attempts to maintain surveillance over another object, or the evader. The Pursuer has a speed advantage over the Evader, but the Evader is more maneuverable in that the Pursuer is restricted in his turning radius, or rate of change of direction, while the Evader is permitted to change his direction of movement instantaneously. The Pursuer is considered to have a "cookie cutter" detection device; that is, surveillance is maintained only as long as the distance from the Pursuer to the Evader is less than the detection range of the device.

The objective of the Pursuer is to maintain the Evader under surveillance as long as possible and conversely the Evader attempts to escape the Pursuer's detection region as quickly as possible.

The following notation is repeated from [1]:

- s_1 = Pursuer's speed with maximum w_1
- s_2 = Evader's speed with maximum w_2
- R = Pursuer's minimum turning radius
- ϕ = Fraction of maximum course curvature employed by Pursuer ($\phi = -1$ corresponds to left turn with minimum turning radius)

ψ = Evader's heading relative to that of Pursuer
 d = Pursuer's detection range
 T = time for Evader to escape (reach circle of radius d from Pursuer)

This study was conducted from the point of view of the Pursuer. Therefore the problem is (from [1])

$$\max \min \int_0^T dt$$

subject to the equations of motion of the Pursuer and the Evader. In solving this problem it is customary to use a relative coordinate system wherein the Pursuer is always at the origin with a heading in the positive direction along the vertical axis. In this way the problem becomes (from [1])

$$\max_{\phi, s_1} \min_{\psi, s_2} \int_0^T dt \quad \text{with } T \text{ unspecified,}$$

$$\text{subject to: } \frac{dx}{dt} = -\frac{s_1 y \phi}{R} + s_2 \sin \psi$$

$$\frac{dy}{dt} = \frac{s_1 x \phi}{R} + s_2 \cos \psi - s_1,$$

$$\text{and } -1 \leq \phi \leq +1,$$

$$0 \leq s_1 \leq w_1,$$

$$0 \leq s_2 \leq w_2 \leq w_1,$$

with initial location of Evader

$$x(t = 0) = x_0,$$

$$y(t = 0) = y_0,$$

and terminal surface defined by

$$x^2(T) + y^2(T) = d^2.$$

For the derivation of these equations the reader is referred to Appendix A of [1].

It is assumed that the situation of perfect information exists; that is, each participant knows the actions of the other instantaneously.

B. OPTIMAL ESCAPE TRAJECTORIES OF THE EVADER

The optimal escape trajectory of the Evader as derived in Appendix D of [1] is given by

$$x(\tau) = (d - \tau w_2) \sin(u + \frac{w_1}{R} \phi \tau) + \frac{R}{\phi} (1 - \cos \frac{w_1}{R} \phi \tau), \quad (1)$$

and

$$y(\tau) = (d - \tau w_2) \cos(u + \frac{w_1}{R} \phi \tau) + \frac{R}{\phi} \sin \frac{w_1}{R} \phi \tau, \quad (2)$$

for

$$U \leq u \leq \frac{\pi}{2}$$

$$\frac{3}{2}\pi \leq u \leq 2\pi - U$$

where

$$\cos U = \frac{w_2}{w_1},$$

$$0 \leq U \leq \frac{\pi}{2},$$

and $\tau = (T - t)$.

U denotes the angle between the heading of the Pursuer and the point of escape from the detection region by the Evader.

III. DISCUSSION OF THE MODEL

Escape trajectories for the case where the model indicated that surveillance could not be maintained were plotted for angles of escape from the detection region of 60 degrees through 180 degrees at 10 degree intervals. The solutions to equations (1) and (2) for value of τ greater than zero established these paths as shown in Figure 1. The values of the parameters used in all computations in this report were:

$$\begin{aligned} R &= 20 \\ d &= 10\pi \\ w_1 &= 20 \\ w_2 &= 10 . \end{aligned}$$

A. OPTIMAL ESCAPE PATHS

From Figure 1 it can be seen that escape paths exist for all points in the state space and the "surveillance pockets" which Taylor [1] suspected are not present. Also, the paths for escape at points between 60 degrees and 90 degrees are smooth and continuous.

It should be noted, however, that although all points on a given curve satisfy equations (1) and (2), only that portion of the curve in the first quadrant of the detection region is optimal.

For the curve BB'B", for example, the value of τ in (1) and (2) which give the point B' is 1.2. However, if at that

point the Pursuer were to stop ($s_1 = 0$), the Evader would escape in the shortest length of time by steering a course directly away from the Pursuer and would escape at point D. An examination of the distance and speed involved in this tactic discloses the fact that the time required to escape from point B' to D is 2.5, which indicates that this is preferred by the Pursuer, who desires to maximize the time until the Evader escapes. That is to say that although the entire curve BB'B" describes an escape route, only that portion from B to B' lies on the optimal path. Extending this results in the conclusion that all optimal escape paths which intersect the radius OD utilize that portion of OD from the point of intersection to point D. Those portions of the curves lying in the fourth quadrant, while constituting escape paths, are not optimal.

Similarly, those curves which intersect the vertical radius OE before reaching the horizontal diameter D'D are also optimal only in the first quadrant. For example, curve CC'C" describes an escape path, but the time to escape, τ , following the curve C'C" is 1.4 while that for the line segment C'OC" is 3.71. At point C' the optimal tactic for the Pursuer is to steer directly toward the Evader ($\phi = 0$) and stop when the Evader is at point O. This will cause the time required to escape from point C' to be as noted above (3.71).

As pointed out by Taylor, for $u = U = 60$ degrees, the path does form a cusp. This cusp occurs at time $\tau = 1.4$ at point A' on Figure 1. That part of the curve for values of

τ greater than 1.4 is not optimal, for an Evader on that portion of the curve could escape more quickly by following a path which would lead him to escape the detection region at some escape angle greater than 60 degrees. That part of the curve for $u = 60$ degrees for values of τ less than 1.4 (arc AA') is indeed the barrier, for those segments of the escape paths (for escape angles greater than 60 degrees) to the right of the barrier are likewise not optimal in that an Evader could escape the detection region sooner by following a path roughly parallel to A'A.

This termination of the barrier at point A' is the criteria which indicates that no surveillance zone exists. If such a zone did in fact exist, the barrier would be the boundary of a closed set of points which would not lie on any escape path. As can be seen from the mapping of paths on Figure 1, such a set of points does not exist.

B. ADEQUACY OF THE MODEL

In examining the requirement of the model that the Pursuer stop ($s_1 = 0$) at the time that the Evader reaches a point 90 degrees or 270 degrees relative to the Pursuer's heading, it was discovered that a clarification of equation (D8) of Appendix D of [1] was required. The equation is repeated here.

$$s_1(t) = \begin{cases} w_1 & \text{for } -\frac{\pi}{2} < u < \frac{\pi}{2} \\ 0 & \text{for } \frac{\pi}{2} < u < \frac{3}{2}\pi \end{cases}$$

It would appear that this equation states that if the escape angle will be greater than 90 degrees the Pursuer should stop. This is incorrect as the value of s_1 is not a function of u , the escape angle, but rather is a function of time and the position of the Evader on the escape path. Since $\tau = (T - t)$, s_1 is further a function of τ . The equation and conditions for its equality are correctly stated as

$$s_1(y(\tau)) = \begin{cases} w_1 & \text{for } \tau \text{ such that } y(\tau) > 0 \\ 0 & \text{otherwise.} \end{cases}$$

The values of the parameters used were considered appropriate for a destroyer-submarine surveillance situation, considering conventional submarines and the sonar equipment most commonly in use in the present-day fleet. For the newer sonar equipments the ratio of sonar range to minimum turning radius of the destroyer would be low, however, but it is felt that this does not significantly detract from the result of this study.

One aspect of the operational situation which is not covered by the model examined is the reacquisition of the Evader after it has escaped from the detection region of the Pursuer. This is an important aspect, for surveillance of potentially hostile submarine contacts calls for extended periods of surveillance, and while this model maximizes the time which is required for the Evader to escape the detection region, it is a "single pass" model.

C. REACQUIRING EVADER AFTER ESCAPE

A model which would be more applicable to a continuous surveillance-evasion situation would be one which would take under consideration the fact that, since no surveillance zone exists, the Evader will eventually escape the detection region and must be reacquired for long-term surveillance to be possible.

Such a model should have as its objective the minimization of reacquisition time; that is, it should minimize the time during which the Evader is not in the detection region of the Pursuer.

The tactics utilized by the Pursuer to place the Evader back in his detection region after escape has occurred would depend on several factors, but primarily they would depend on the position of the Evader at the time of escape and the assumptions concerning perfect or imperfect information.

Concerning the latter, the assumption of imperfect information once escape has occurred would be valid from the viewpoint of the Pursuer, since the Evader is no longer within the detection region of the Pursuer, but would be less valid from the point of view of the Evader if one is considering a destroyer-submarine situation. This is true because of the much longer range capability of the passive detection devices in use aboard submarine type vessels as compared to the active ranging devices commonly in use aboard the destroyer type ships.

Conversely, the assumption of perfect information once escape has occurred is invalid from the point of view of the Pursuer (the Evader is no longer in the detection region of the Pursuer) but is more valid when considering the Evader, for the same reason mentioned above.

Therefore, the more realistic approach would be to assume perfect information to the Evader on the movements of the Pursuer, but imperfect information to the Pursuer on the movement of the Evader.

A further assumption of instantaneous acceleration and deceleration for both Pursuer and Evader, while not completely realistic, is in order and it is felt that this would have little or no effect on the accuracy of the models.

It has been noted that, given optimal maneuvering by both Pursuer and Evader, escape will occur at a point on the boundary of the detection region at an escape angle u , such that (for the particular parameters used in this study)

$$90^\circ \geq u \geq 60^\circ$$

for those escape paths which do not intersect the vertical axis of the detection region (radius OE in Figure 1) and at $u = 180$ degrees for those paths which do intersect the vertical axis.

Consider the possible actions for the Pursuer at the time of escape in the first case. One such action would be to continue in a turn ($\phi = +1$) at maximum speed ($s_1 = w_1$), in which case the Pursuer's trajectory would be as described by

the arc AA' in Figure 2. Denote the time required for the Pursuer to travel the arclength from A to A' by t . At the instant the Pursuer arrives at point A' , the Evader must be within a circle of radius Z centered at point B , point B representing the point at which the Evader escaped from the detection region, where $Z = w_2 t$. The probability that the Pursuer is again maintaining surveillance of the Evader when at point A' is represented by the ratio of the area of the intersection of the circles centered on A' and B to the area of the circle of radius Z centered on point B . If the Evader is within the detection region at this time, the escape paths of Figure 1 can be used to again effect an escape.

A second possible course of action would be to maneuver as indicated in Figure 3. In this case, the initial action at the time of escape would be a turn to the left ($\phi = -1$) at point A , with a subsequent turn to the right at point A' ($\phi = +1$) to traverse an arc $A'B$ which would place the Pursuer at point B , the last known position of the Evader, at time t . Again, due to the speed restriction on the Evader, the Evader will be within a circle of radius Z centered at point B , where again $Z = w_2 t$. The probability that the Evader is under surveillance at time t is again represented by the ratio of the area of possibility of the Evader which is enclosed in the detection region of the Pursuer to total area of possibility of the Evader.

For the case of escape from the detection region at 180 degrees, it is proposed that a turn to the right at the time of escape ($\phi = +1$) at maximum speed ($s_1 = w_1$) be made by the Pursuer at point A in Figure 4. This turn would be maintained until the Pursuer arrived at point A', which is the point of tangency with the turning circle of a tangent through the last known position of the Evader, point B. As can be determined by examination of the radius of possibility of the Evader, the probability that the Evader is within the detection region when the Pursuer arrives at point B is quite low.

It should be noted that the optimal values of τ as seen on Figure 1 indicate that the surveillance time on each pass of the "single pass" model examined is greater for those points on the detection circle near the heading of the Pursuer than for points at larger angles from the Pursuer's heading. Therefore, it would appear that it would be advantageous to attempt to maneuver the Pursuer to enable him to reacquire the Evader at a point as near as possible to the Pursuer's heading. This would appear to be possible as the assumption of perfect information on the part of the Evader would permit one to predict the Evader's movements during the time he is out of the detection region of the Pursuer.

IV. CONCLUSIONS

As a model to describe a surveillance-evasion situation, the differential game model examined proved to realistically describe a situation which would confront a destroyer attempting to maintain surveillance over a potentially hostile submarine contact. However, the model fails to provide for a long term surveillance situation which is currently required to be met by the operating forces at sea.

In the case whereby the Evader can escape the detection region, the current model must be modified to provide for reacquisition after the escape or it must be used in conjunction with some existing pursuit-evasion model. The suggestions provided in this paper hopefully will provide insight into the problems surrounding reacquisition and a basis for further research effort in this vital area.

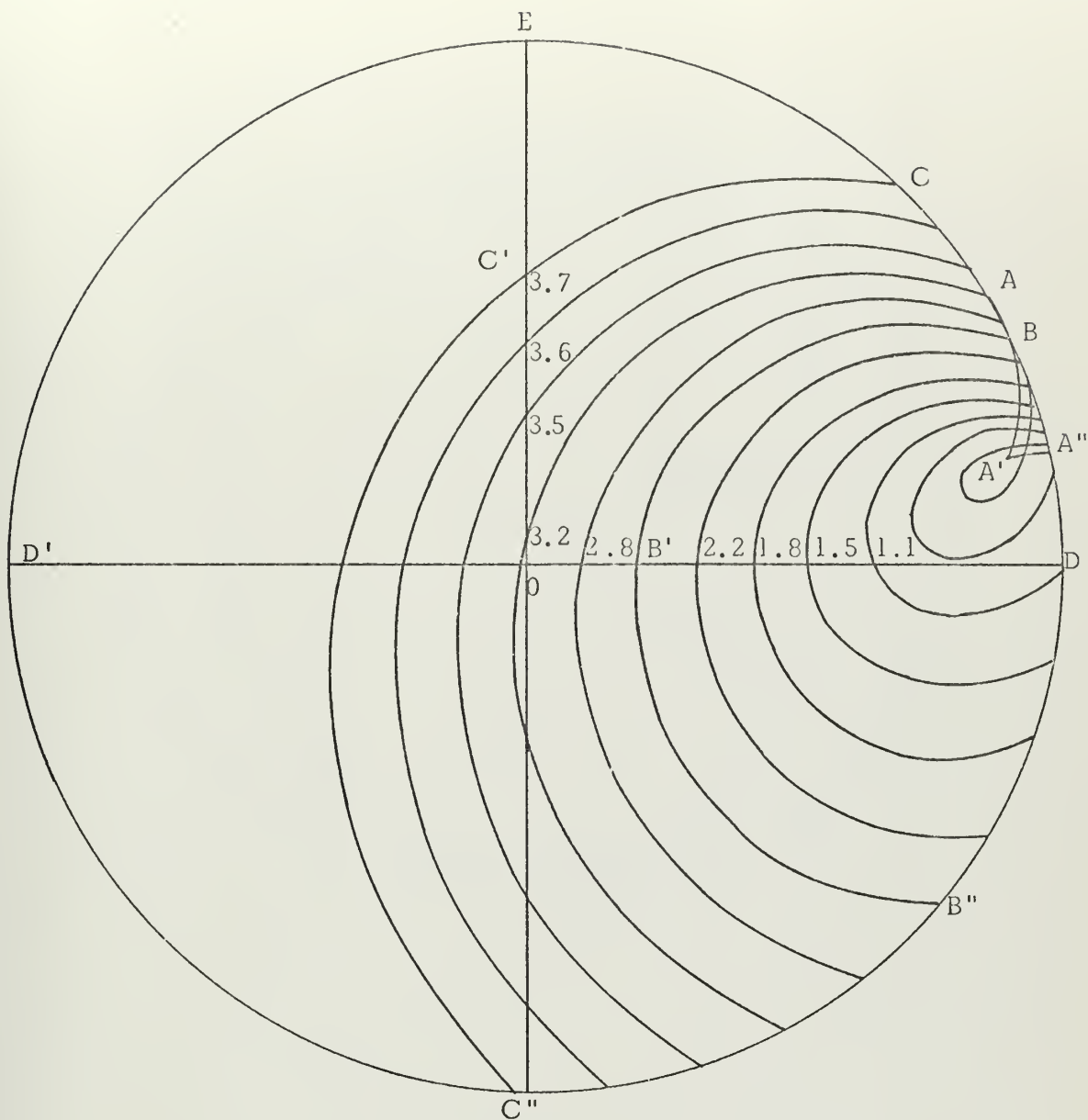


Figure 1. Escape Paths. Figures denote time until escape.

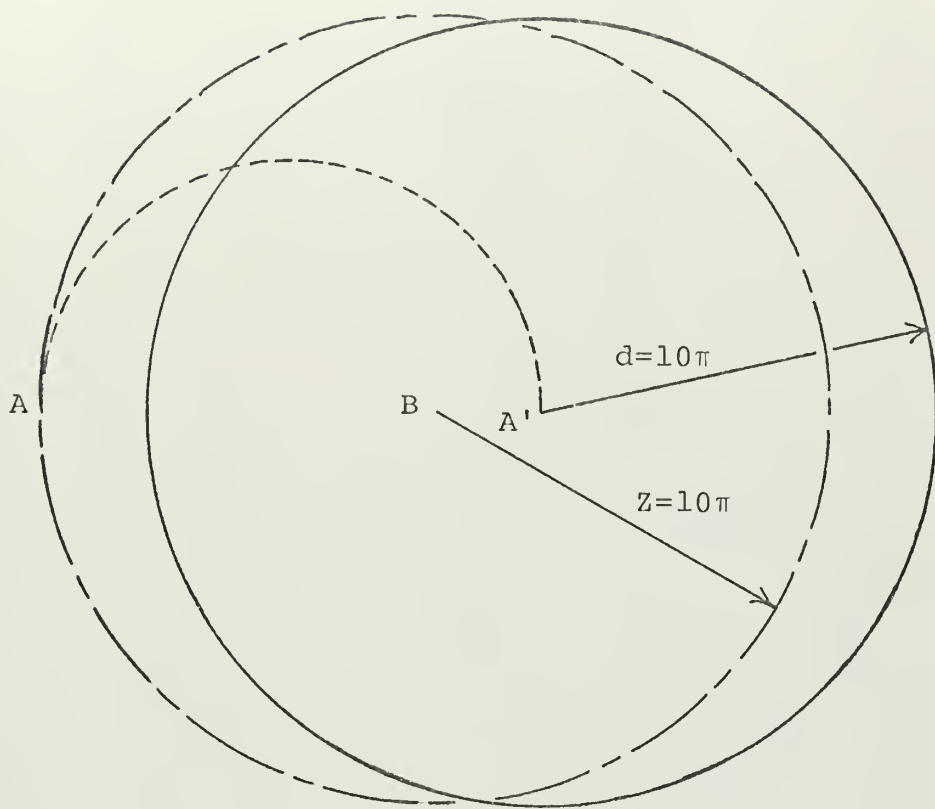


Figure 2. Maneuver for Reacquisition for 90 degree escape angle.

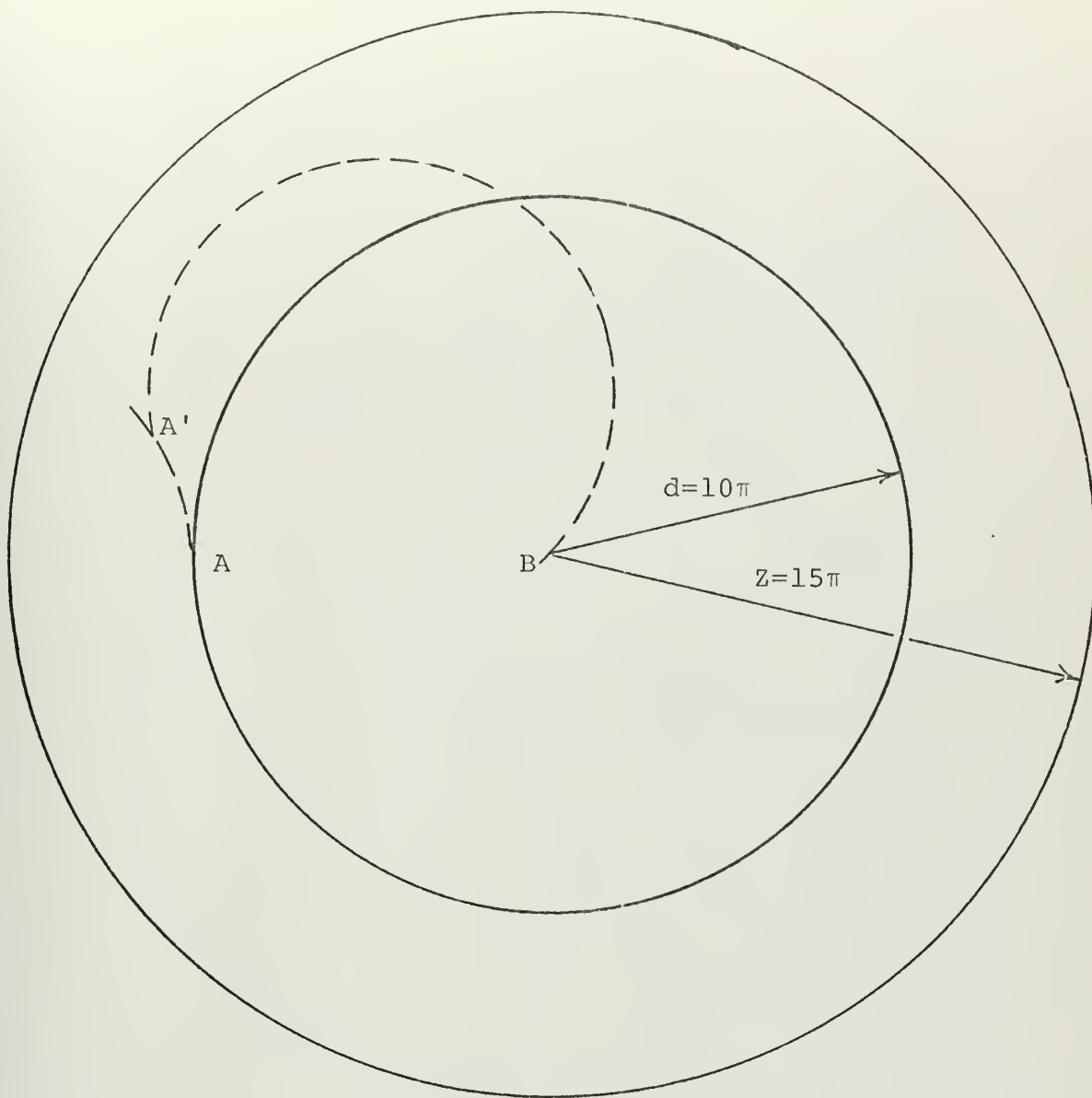


Figure 3. Alternate Manuever--90 degree escape angle.

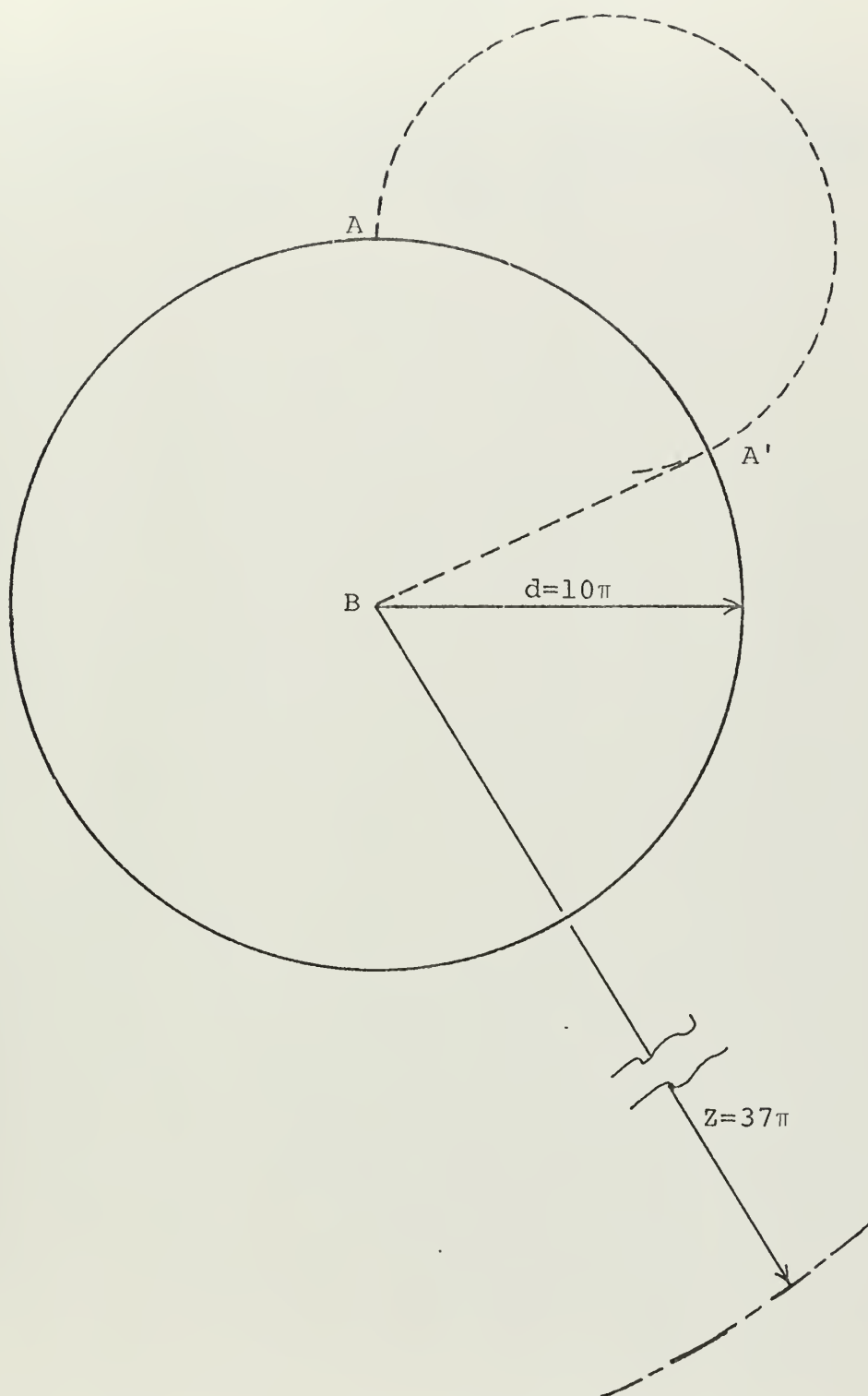


Figure 4. Maneuver for Reacquisition for 180 degree escape angle.

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1. J. Taylor, "Application of Differential Games to Problems of Naval Warfare: Surveillance-Evasion--Part I," Naval Postgraduate School, Monterey, California 93940, 19 June 1970.

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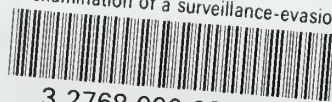
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